Courtney Primary School

Calculation Policy

September 2021

The National Curriculum 2014

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. … pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

The national curriculum for mathematics aims to ensure that all pupils:

• become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately;

• **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language;

• can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Introduction to Calculations

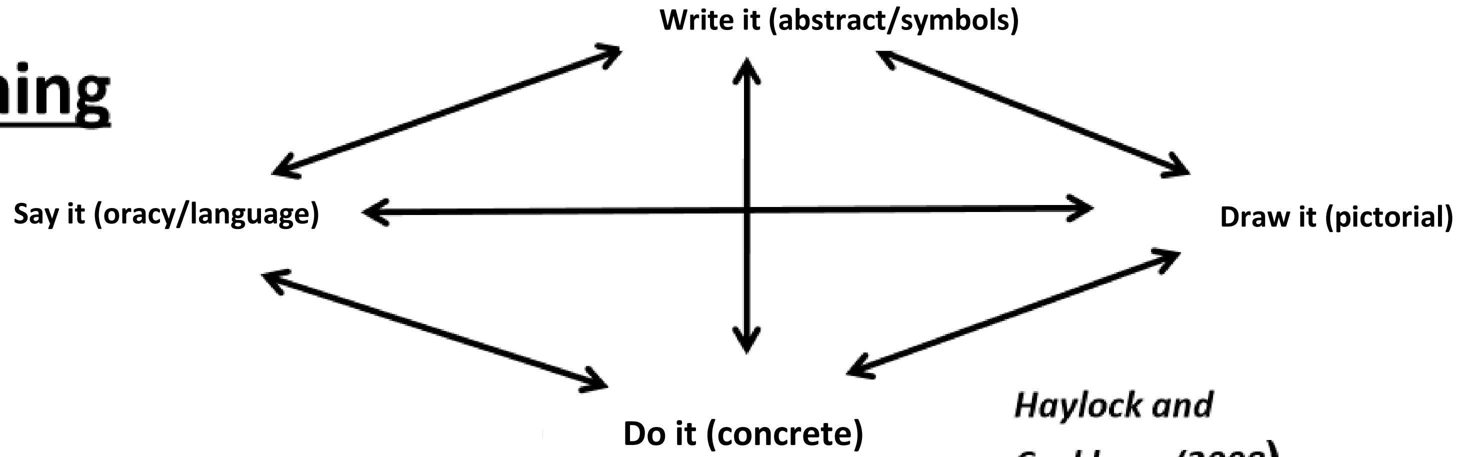
Written methods of calculation are based on mental strategies. Each of the four operations build on mental skills which provide the foundation for jottings and informal written methods of recording. It is essential that these skills are refined before leading on to more formal written methods of calculation. See the appendix ‘Fluency Progression – Expected Number Facts’ to see which mental methods are covered in each year group.

Strategies for calculation always need to be represented by models and images to support, develop and secure conceptual understanding. This will build a deeper understanding and fluency. When teaching a new strategy, it is important to start with numbers that the child can easily manipulate so that they can understand the methodology. Also, where possible, make links to concrete resources and stem sentences used in previous year groups so children can see how the methods are linked.

The transition between stages should not be hurried. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

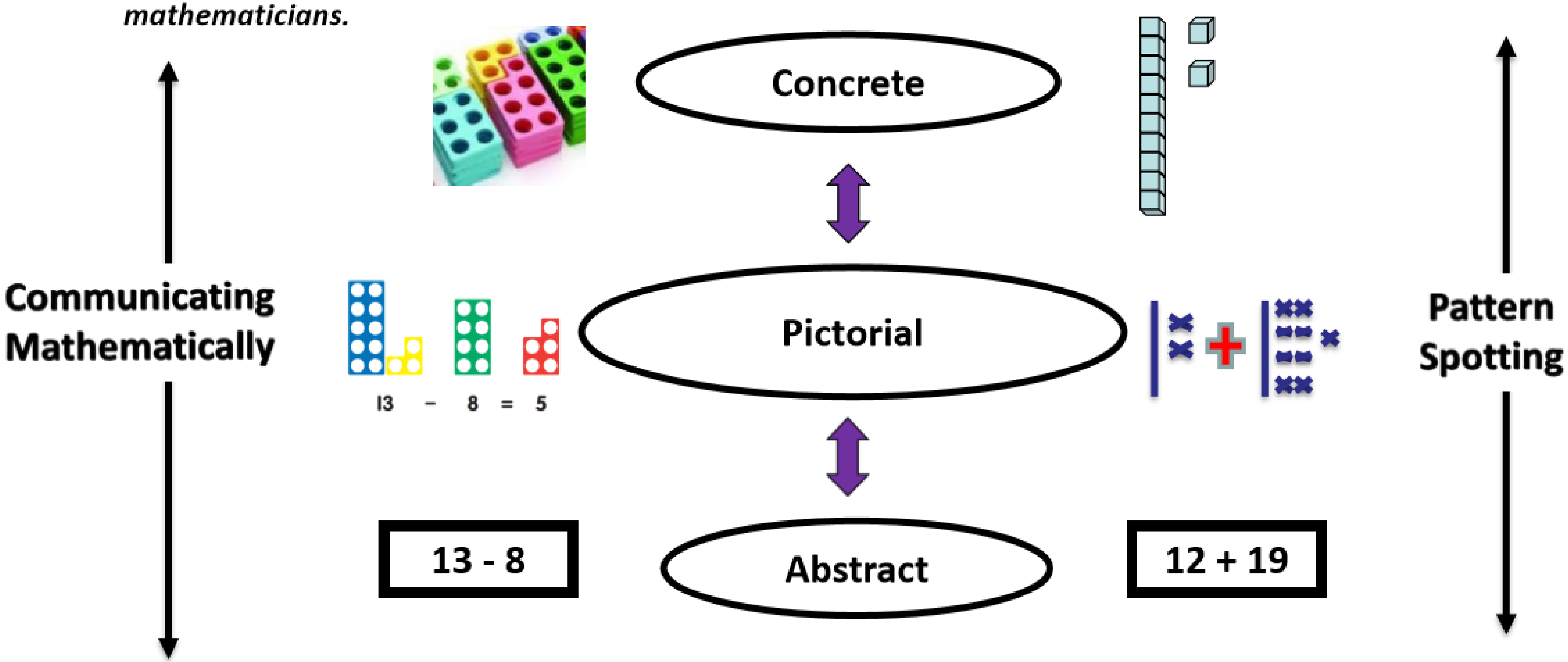
A sound understanding of the number system (place value) is essential for children to carry out calculations efficiently and accurately.

Structuring Learning



Haylock and Cockburn (2008)

Children must have concrete experiences that enable them to create visual images. They should be encouraged to articulate their learning and to become pattern spotters. This will enable them to become able mathematicians.



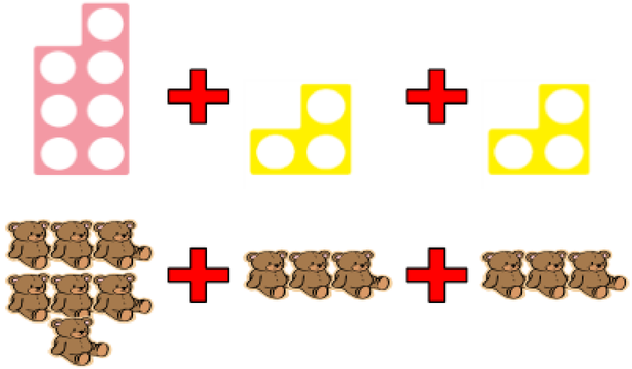
Addition

Structures of Addition (Haylock and Cockburn 2008)

Children should experience problems with all the different addition structures in a range of practical and relevant contexts e.g. money and measurement

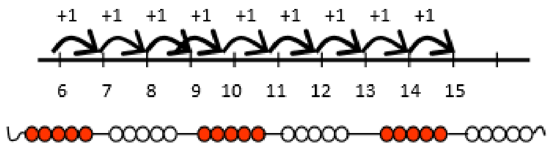
Aggregation

Union of two sets
How many/much altogether?
The total



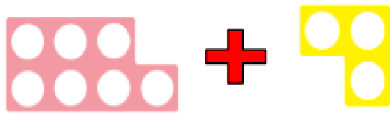
Augmentation

Start at and count on
Increase by
Go up by

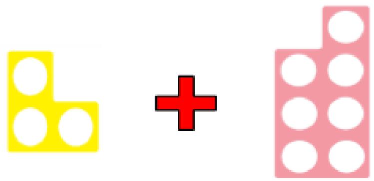


Commutative law

Understand addition can be done in any order
Start with bigger number when counting on
(Explain to children that subtraction does not have this property)

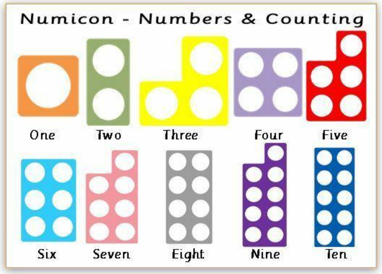


is the same as/equal to (=)



Addition Reception

Before addition can be introduced, children in Reception build on concepts taught in nursery/pre-school by working through the maths in Ranges 5 and 6 of 'Birth to 5 Matters'. Children need to have a secure knowledge of number in order to begin addition. Lots of objects (including Numicon, Numberblocks, tens frames and fingers) are used indoors and outdoors to teach numbers to 5 and then numbers to 10.

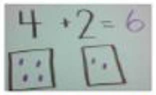


Children are then introduced to the concept of addition as 'number bonds' through practical games and activities. In particular, children learn **number bonds up to 5 and some number bonds to 10** so that ultimately they can recall these facts automatically. Children act out addition sums to physically add two groups of objects together and use arm gestures to represent the signs + and =. This is reinforced by opportunities provided in the outdoor area for the children to use addition e.g. adding together groups of building blocks, twigs etc. Children build on their previous knowledge of 'more' by learning that adding two groups of objects together gives them a larger number (more objects). Adults model addition vocabulary supported by age appropriate definitions. An example of this is, "Addition means we add two groups together / we put two lots of objects together. Equals means we find out how many we have got altogether. 3 add 2 equals 5! We have got 5 altogether." Adults support children in recording their addition sums using their own jottings on whiteboard and paper, and, if appropriate, using the formal written equation.

Do it (concrete) / Draw it (pictorial) / Write it (abstract)

Children can begin to combine groups of objects using concrete apparatus:

Construct number sentences verbally or using cards to go with practical activities.

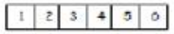


Children are encouraged to read number sentences aloud in different ways "Three add two equals 5" "5 is <u>equal</u> to three and two" "5 is the <u>same as</u> three and two"

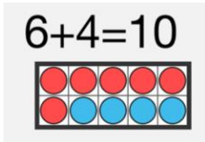
Children make a record in pictures, words or symbols of addition activities.

Solve simple problems using fingers

Number tracks can be introduced to count up on and to find one more:



What is 1 more than 4? 1 more than 13?



Say it (oracy)

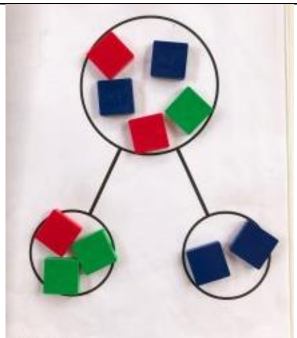
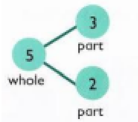
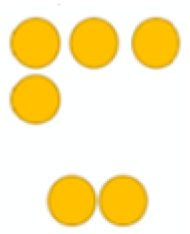
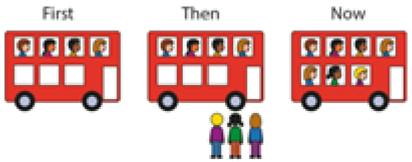
There are more ____ than _____

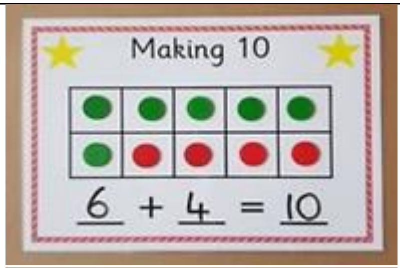
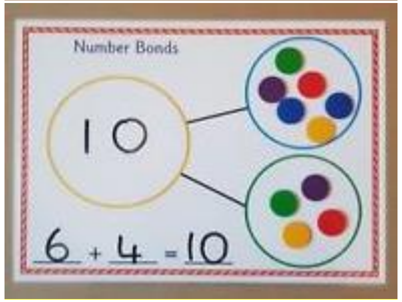
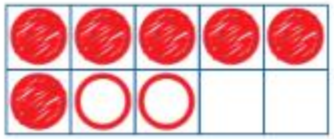
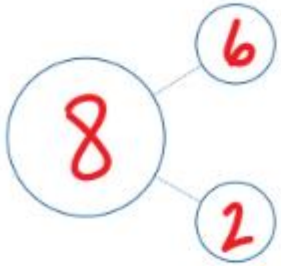
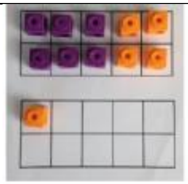
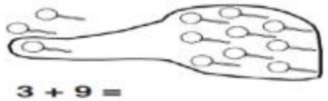
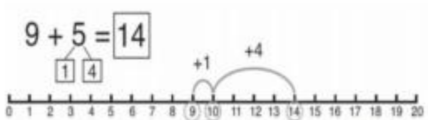
One more than __ is __.
___ is one more than ___.
Adding 1 gives 1 more.

When zero is added to a number, the number stays the same.

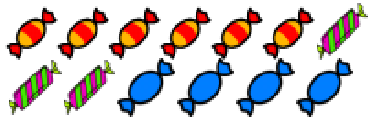
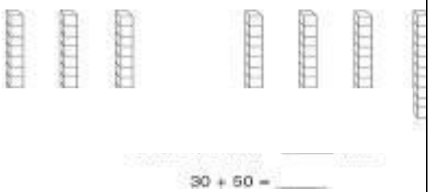
This is a whole ____, because I have all of it.
This is not a whole ______, because I only have part of it.
A whole is always bigger than part of a whole.
A part is always smaller than its whole.

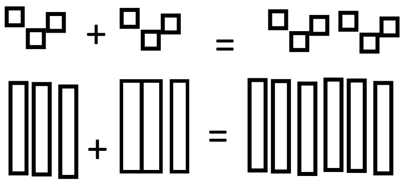
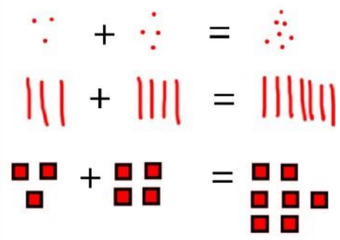
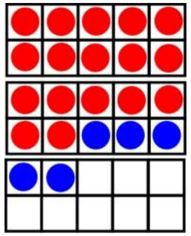
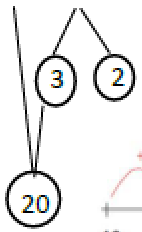
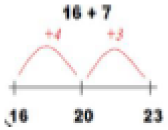
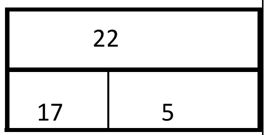
Addition Year One

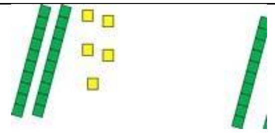
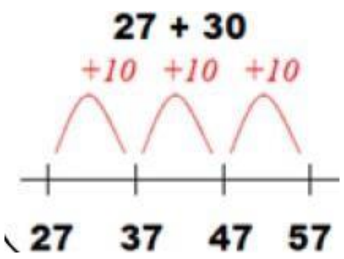
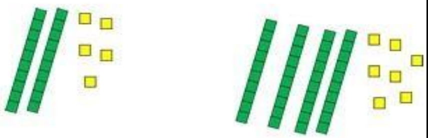
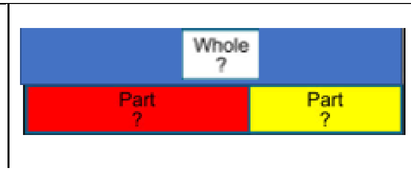
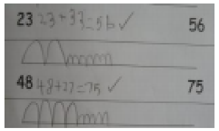
Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)	Say it (oracy)
Combining 2 parts to make a whole Counting sets of objects, combining then recounting using a 1:1 correspondence.		  	4 + 3 = 7 I have 4 apples and I pick 3 more, how many have I got altogether?	This is a whole ____, because I have all of it. This is not a whole ______, because I only have part of it. A whole is always bigger than part of a whole. A part is always smaller than its whole. __ is the whole; __ is a part and is a part. This number represents ____ *(the blue cars for example)*
Augmentation (addition) Pupils should learn the structure “First…then…now” within a story representation to embed the understanding that addition is when a quantity increases.	Use physical objects such as counters or cubes to tell augmentation ‘stories’, e.g. “First I have 4 counters then I get two more. Now I have 6 counters.” 	Children may draw pictures or draw counters to show the calculations they are doing. They may progress to drawing bar models or filling in part-part-whole diagrams. 		First…Then…Now There are __ and __. We can write this as __ plus __. The __ represents the __ and the represents the __.

Strategies within 10 Pupils learn that 10 can be partitioned into pairs of numbers that sum to 10. Similarly, then learn the number bonds for all single digit numbers. *See fluency expectations for Y1.*	 	 	0+10=10 0+7=7 1+9=10 1+6=7 2+8=10 2+5=7 3+7=10 3+4=7 4+6=10 4+3=7 5+5=10 5+2=7 6+4=10 6+1=7 7+3=10 7+0=7 8+2=10 9+1=10 10+0=10	If we change the order of the addends, the sum remains the same. Because addition is commutative we can add two numbers in any order and the sum remains the same.
Regrouping to make 10 Pupils use their number bond knowledge and bridge to 10 e.g.	 Use tens frames to demonstrate that the sum is larger than ten.	 	7p+4p=11p I have 7p, how much more do I need to make 10p. How much more do I add on now? If you know 10=7+3, what else do you know?	There are ___, ___ and ___. Altogether there are____. __ is __ ones __ is ___ tens and __ ones

Addition Year Two

Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)	Say it (oracy)
Adding 3 single digits Use this method as an opportunity to develop fluent recall and application of known number facts including bonds and doubles.	  *Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.* Consider other strategies too. Are there other number bonds they know? Doubles? Etc.	 **6 + 3 + 4 = 13** 13 6 | 3 | 4	$\underbrace{4 + 7 + 6}_{10} = \boxed{10} + \boxed{7}$ $= \boxed{17}$	When we add three numbers, the total will be the same whichever pair we add first. If you change the order of the addends, the sum stays the same. First...then...then...now.... We can look for pairs of addends which sum to 10. __ plus __ is equal to ten, then ten plus___ is equal to___.
Adding multiples of ten	50= 30 = 20  Model using dienes and bead strings	 Use representations for base ten. Practise drawing dienes.	20 + 30 = 50 70 = 50 + 20 40 + □ = 60	There are ___tens, ___ and ___tens . Altogether there are ___tens.

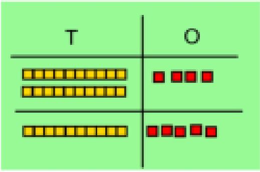
Using known facts	Use dienes/base ten to model the relationships. 	 Children draw representations of H,T and O	3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* 300 + 400 = 700	I know ____ plus ___ is equal to ___, so ____ tens plus ____ tens is equal to ____ tens. I know ____ plus ___ is equal to ___, so ____ hundreds plus ____hundreds is equal to ____ hundreds. I know that _ plus _ is equal to ten. So _ tens plus _ tens is equal to ten tens. ._ plus _ is equal to one hundred. If I know ___, then I know ____.
Adding a two-digit number and ones	 17 + 5 = 22 Use ten frame to make 'magic ten' Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	Use part-whole, and number line to model. 17 + 5 = 22  	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17 	There are ___, ___ and ___. Altogether there are____. __ is __ ones __ is ___ tens and __ ones When we add and multiply whole numbers our answer gets bigger. When we take away and divide whole numbers our answer gets smaller.

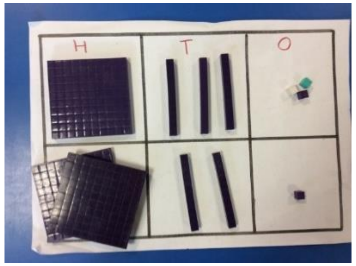
Adding a two-digit number and tens	 25 + 10 = 35 Explore that the ones digit does not change.	 	27 + 10 = 37 27 + 20 = 47 27 + □ = 57	When we find ten more, the tens digit changes and the ones digit stays the same. Ten more than __ is ___. ____ is ten more than ___. We had __ tens and __ ones. Ten more gives us __ tens and _ones.
Adding two two-digit numbers Partition the numbers to show what the digits mean. The emphasis for this strategy in KS1 is to develop a deep understanding of place value. In year 2, recording addition and subtraction informally in columns supports place value and prepares for formal written methods with larger numbers later on in KS2 . Ensure that when moving into any form of column the ones are calculated first.	  Model using dienes , place value counters and numicon	 After practically using the base 10 blocks/dienes, draw the dienes for the calculations: • lines for tens • dots for ones	25 + 47 20 + 5 40 + 7 20 + 40 = 60 5+ 7 =12 60 + 12 = 72  Counting on in tens and ones to solve missing number problems	There are... and... We can write this as___plus____. The ____ represents the ... ___ is equal to __ plus ___. ___ plus ___ is equal to ___. ___ and ____ are the addends. ___ is the sum. Stem sentences to remember the steps in the method.

Addition Year Three

<table>
<tr><th>Strategies</th><th>Do it (concrete)</th><th>Draw it (pictorial)</th><th>Write it (abstract)</th></tr>
<tr>
<td>Column method- no regrouping

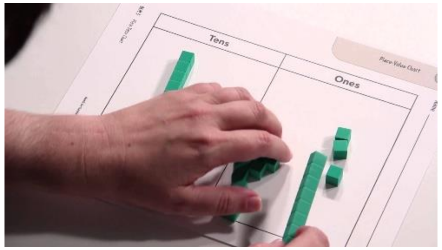
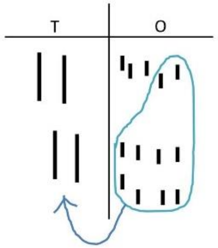
Use this for adding numbers with two or three digits.</td>
<td>24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks.




</td>
<td>After practically using the base 10 blocks/dienes, draw them for the calculations:
squares for hundreds
lines for tens
dots for ones</td>
<td><u>Calculations</u>

21 + 42 =

T O
21
+ <u>42</u></td>
</tr>
</table>

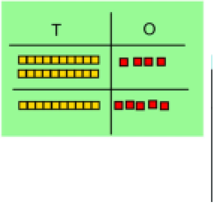
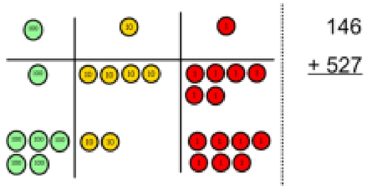
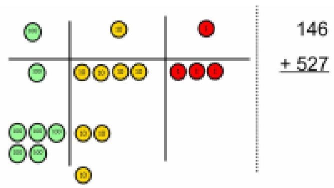
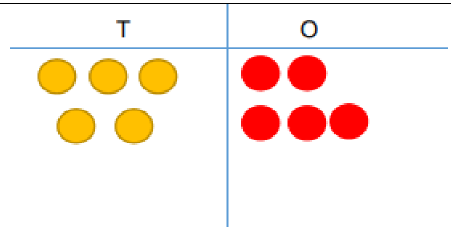
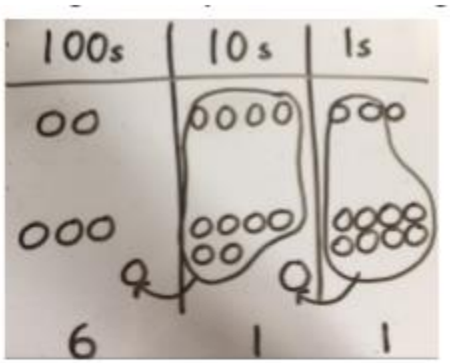
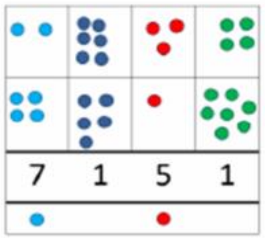
Column method – regrouping	Make both numbers on a place value grid using base ten/dienes, e.g. 28 + 15. Add up the ones and show how you can regroup ten ones into one ten: 	Children can draw pictorial representations of the base ten/dienes to further support their conceptual understanding:  $\begin{array}{r} 25 \\ +\underline{18} \\ \underline{43} \\ 1 \end{array}$	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. $\begin{array}{lll} 20 & + & 5 \\ \underline{40} & \underline{+} & \underline{8} \\ 60 & + & 13 \quad = 73 \end{array}$ $\begin{array}{r} 536 \\ \underline{+\ 85} \\ \underline{621} \\ 1\,1 \end{array}$ Once children are confident with the column method, introduce examples with missing digits.

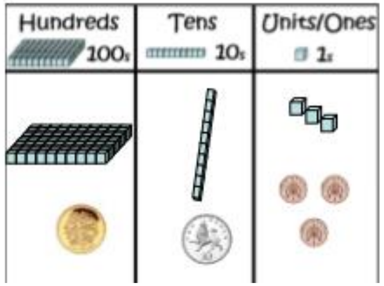
Say it (oracy)

___ is equal to __ plus ___.
___ plus ___ is equal to ___.
Addend plus addend is equal to the sum.
We line up the ones; __ ones plus __ ones.
We line up the tens; __ tens plus __ tens.
The __ is in the ones column - it represents__ ones.
The __ is in the tens column - it represents __ tens.
In column addition, we start at the right-hand side.
__ ones plus __ ones is equal to __ ones.
__tens plus __ tens is equal to __ tens.

If the column sum is equal to ten or more, we must exchange the ten from one column to the next place value column.

Addition Year Four

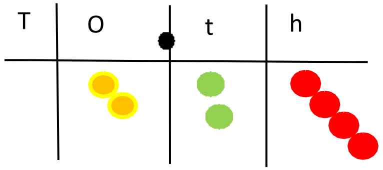
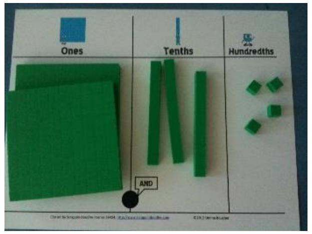
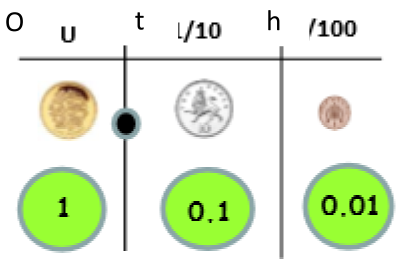
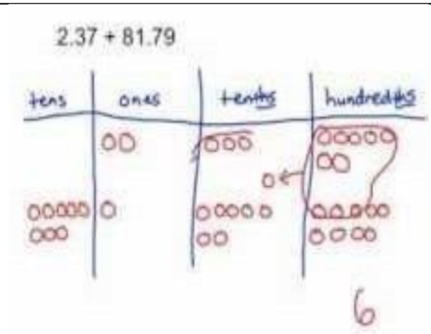
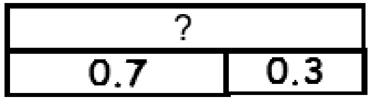
Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)
Column method up to four digits Add numbers with up to **four digits**, using formal written methods of columnar addition. Estimate the answer to a calculation and use inverse operations to check answers. Solve addition two step problems in contexts, deciding which operations and methods to use and why. **Ensure children are provided with opportunities to add more than 2 numbers.**	As year three but increasing to four digit numbers. Initially re-cap base ten and then move on to using place value counters.  24 + 15= 39 Make both numbers on a place value grid.  Add up the ones and model regrouping so that ten ones become one ten  Add up the rest of the columns, exchanging the 10 counters from one column for the next place value colum until every column has been added.	 After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.  	Calculations 21 + 42 = $\begin{array}{r} 21 \\ +\ \underline{42} \end{array}$ 1845 + 526 = 2371 $\begin{array}{r} 1845 \\ +\ \underline{526} \\ \underline{2371} \\ {\scriptstyle 1\ \ 1} \end{array}$ 1845 + $\square$ = 2371

	Also add decimals in the context of money. Again, link coins to base ten/dienes to ensure conceptual understanding:  		

Say it (oracy)

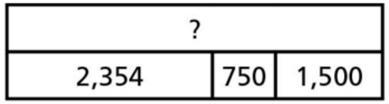
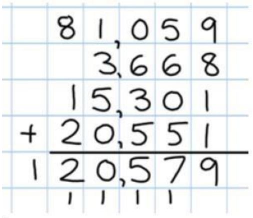
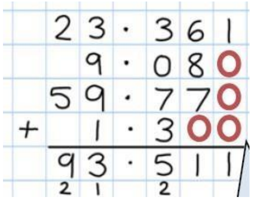
___ is equal to __ plus ___.
___ plus ___ is equal to ___.
Addend plus addend is equal to the sum.
We line up the ones; __ ones plus __ ones.
We line up the tens; __ tens plus __ tens.
The __ is in the ones column - it represents__ ones.
The __ is in the tens column - it represents __ tens.
In column addition, we start at the right-hand side.
__ ones plus __ ones is equal to __ ones.
__tens plus __ tens is equal to __ tens.

Addition Year Five

Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)
As Year four but progressing to more than 4 digits. Add decimals to two decimal places, including money.	 Introduce decimal place value counters and model exchange for addition. For decimals, ensure children have an understanding of the size. This can be done using measure (e.g. 1m, 1cm, 1mm), by physically cutting ribbon/paper/straws into ten equal pieces or using base ten/dienes:  	 Bar Model 	72.8 + 54.6 127.4 1 1 £ 2 3 · 5 9 + £ 7 · 5 5 £ 3 1 · 1 4 21,848 + 1,523 = 23,371 21848 + 1523 23371 1 1 Ensure children insert zeros as place holders. £154.75 + £233.82 = £388.57 154·75 + 233·82 388·57 1 21,848 + $\boxed{}$ = 23,371

Say it (oracy)
___ is equal to __ plus ___. ___ plus ___ is equal to ___. Addend plus addend is equal to the sum. We line up the ones; __ ones plus __ ones. We line up the tens; __ tens plus __ tens. The __ is in the ones column - it represents__ ones. The __ is in the tens column - it represents __ tens. In column addition, we start at the right-hand side. __ ones plus __ ones is equal to __ ones. __tens plus __ tens is equal to __ tens.

Addition Year Six

Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)
As Y5 but developing to add more digits. Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy. **Ensure children are provided with opportunities to add more than 2 numbers.**	As Y5	As Y5 	 Insert zeros for place holders. Including adding money, measure and decimals with different numbers of decimal points.

In Year Six, children should be taught to solve multi-step calculations involving more than one operation – mentally and using the written methods they have been taught – including numbers with more than four digits.

Say it (oracy)

___ is equal to __ plus ___.
___ plus ___ is equal to ___.
Addend plus addend is equal to the sum.
We line up the ones; __ ones plus __ ones.
We line up the tens; __ tens plus __ tens.
The __ is in the ones column - it represents__ ones.
The __ is in the tens column - it represents __ tens.
In column addition, we start at the right-hand side.
__ ones plus __ ones is equal to __ ones.
__tens plus __ tens is equal to __ tens.

Subtraction

Structures of Subtraction (Haylock and Cockburn 2008)

Children should experience problems with all the different subtraction structures in a range of practical and relevant contexts e.g. money and measurement

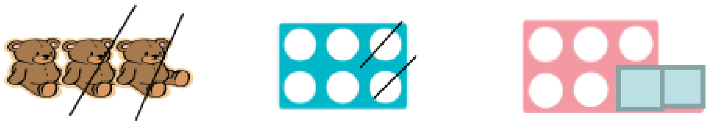
Partitioning

Take away

… how many left?

How many are not?

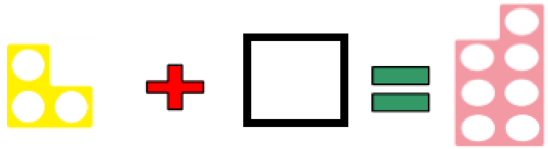
How many do not?

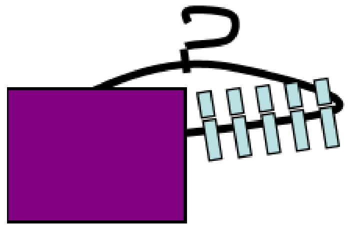


Inverse-of-addition

What must be added?

How many (much) more needed?





There are ten pegs on the hanger – how many are covered?

Comparison

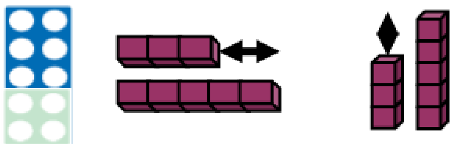
What is the difference?

How many more?

How many less (fewer)?

How much greater?

How much smaller?



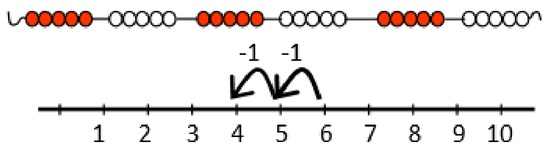
'two more than three is five or two less than five is three'

Reduction

Start at and reduce by

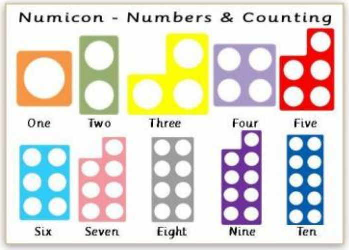
Count back by

Go down by



Subtraction Reception

Before subtraction can be introduced, children in Reception build on concepts taught in nursery/pre-school by working through the maths objectives in Ranges 5 and 6 of 'Birth to Five Matters'. Children need to have a secure knowledge of number in order to begin subtraction. Lots of objects (including Numicon, Numberblocks, tens frames and fingers) are used indoors and outdoors to teach numbers to 5 and then numbers to 10.



Children are then introduced to the concept of subtraction through practical games and activities. In particular children learn **subtraction facts linked to the number bonds up to 5** so that ultimately they can recall these facts automatically. Children act out subtractions to physically subtract a number of objects from a group. Children use arm gestures to represent the signs - and =. This is reinforced by opportunities provided in the outdoor area for the children to subtract e.g. building blocks, twigs etc. Children build on their previous knowledge of 'fewer' and 'less' by learning that subtracting means taking away a certain number of objects from a group (leaving them with fewer objects). Adults model subtraction vocabulary supported by age appropriate definition. An example of this is "Subtraction means we take away objects from a group / we have 11 got fewer objects now. Equals means we find out how many we have got left. Wow! We have only got 3 left!" Adults support children in recording their subtractions using jottings on whiteboards and on paper, and, if appropriate, using the formal written equation.

Do it (concrete) / Draw it (pictorial) / Write it (abstract)

Children begin with mostly pictorial representations or real contexts.

Concrete apparatus is used to relate subtraction to taking away and counting how many objects are left.

Concrete apparatus models the subtraction of 2 objects from a set of 5.

Construct number sentences verbally or using cards to go with practical activities.

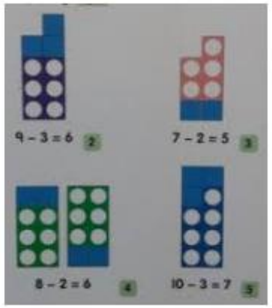
● ● ● ● ✖
5 - 1 = 4

Children are encouraged to read sentences aloud in different ways "five subtract one leaves four" "four is equal to five subtract one" "four is the same as five subtract one"

Children make a record in pictures, words or symbols of subtraction activities.

Solve simple problems using fingers

Number tracks can be introduced to count back and to find one less: What is 1 less than 9? 1 less than 20?

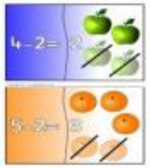
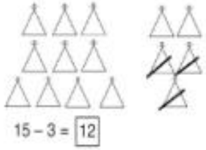
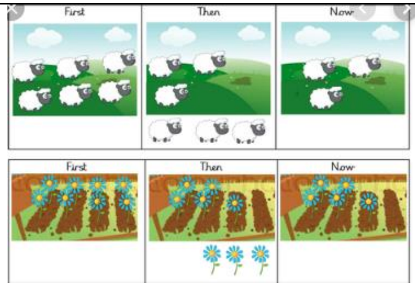


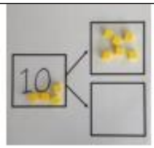
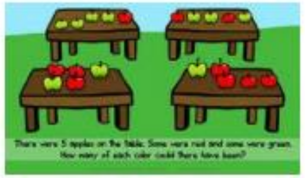
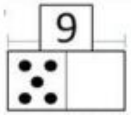
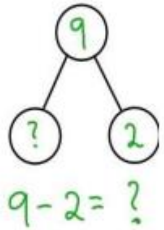
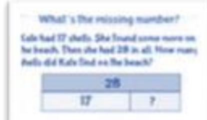
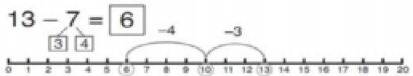
Say it (oracy)

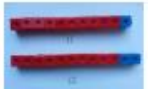
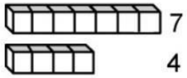
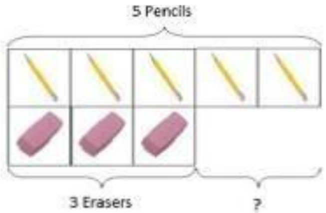
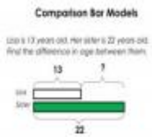
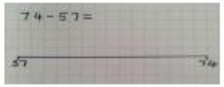
There are fewer__ than __.
One less than __ is __.

This is a whole ____, because I have all of it.
This is not a whole ______, because I only have part of it.
A whole is always bigger than part of a whole.
A part is always smaller than its whole.

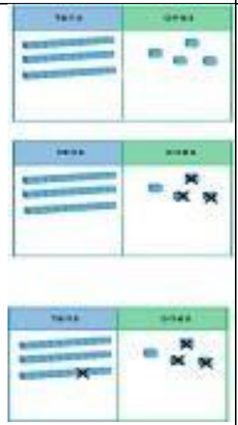
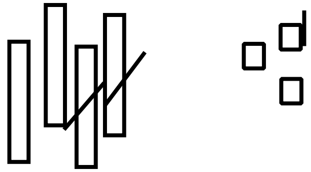
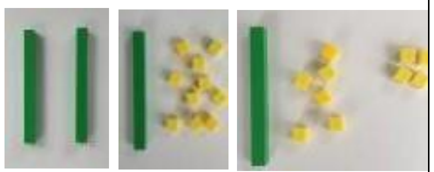
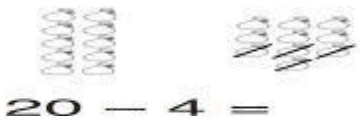
Subtraction Year One

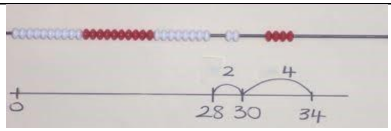
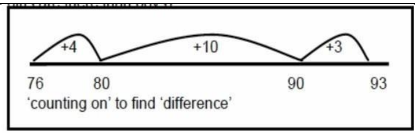
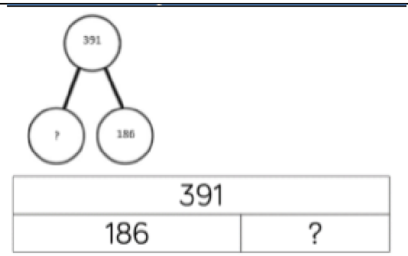
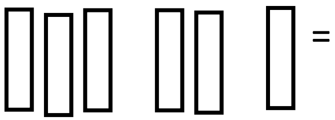
Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)	Say it (oracy)
Taking away ones Use physical objects to demonstrate how something can be taken away. Move on to crossing out drawn representations. This can be developed by representing a group of ten with a line and ones with dots.	  	   23 - 1 = 12	18 -3= 15 8 – 2 = 6 There are 15 cakes in the shop. One cake is eaten, how many are left. Remember to include missing number problems, 7=□-9	Subtracting 1 gives 1 less. Consecutive numbers have a difference of 1. When zero is subtracted from a number, the number stays the same.
Reduction Pupils should learn the structure “First…then…now” within a story representation to embed the understanding that subtraction is when a quantity reduces.	 Use counters or objects and move away from the group as they are counted.		6-3=3 First there were 6 sheep. Then three went away. Now there are 3 sheep.	First… Then...Now...

Part, part, whole model This model develops knowledge of the inverse relationship between addition and subtraction and is used to find the answer to missing number problems.	 If 10 is the whole and 6 is one of the parts. What is the other part? 	  Children should be taught the skills to approach problems in a systematic way. 10 6	 I made 9 buns for the cake sale and I only had 2 left at the end. How many did I sell? 	This is a whole _____, because I have all of it. This is not a whole _______, because I only have part of it. A whole is always bigger than part of a whole. A part is always smaller than its whole. There are __ ___ in the whole group There are ___ __ in this part of the group.
Make 10 Use this strategy to subtract a single digit number from a 2-digit number. Pupils identify how many need to be taken away to make ten first. Then they take away the rest to reach the answer.	 14-5=9 Make 14 on the ten frame or with different coloured cubes to represent the ten and the ones. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.		15 – 7= How many do we subtract to reach the next 10? How many do we have left to subtract?	When taking a single number from a 2 digit number I first take away to make 10. Then I take away the rest.

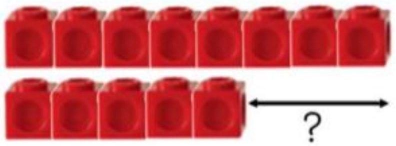
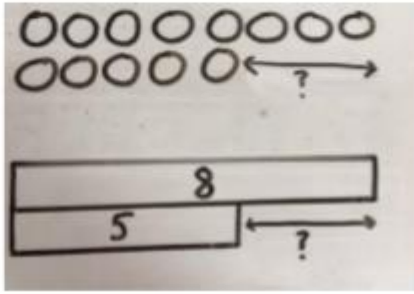
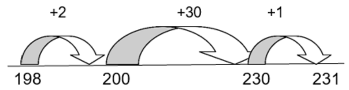
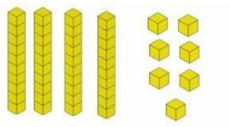
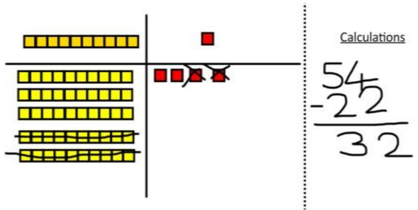
Find the difference Pupils should develop a good understanding of the meaning of 'difference', exploring the inverse relationship with addition by counting back and counting up.	 Practical resources to visualise 'difference' and recognise inverse relationships e.g. 12-1=11 and 11+1=12  Compare objects and amounts  'Seven is 3 more tha 'I am 2 years older t sister'  Lay objects to represent bar model.	  Use a blank number line to count back and count up between 2 numbers.	Lexie has 5 more strawberries than Jake. Jake has 11 cherries. How many does Lexie have? Look at the graph. Fewer children have green eyes than blue. What is the difference?	The __ represents the number of children__. The __ represents the __. The __ represents the difference; it is how many more __ I need.

Subtraction Year Two

Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)	Say it (oracy)
Partitioning to subtract without regrouping The emphasis for this strategy in KS1 is to develop a deep understanding of place value. When not regrouping, partitioning should be developed as a mental strategy where possible, once the concept is understood.	34—13 = 21  Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off.  43—21 = 22 Part whole models Bar models	There are 35 children in the class and 12 are boys. How many are girls? 35-12= 43—21 = 22	The __ represents all the ___. The minus represents the ___. The ___ represents the difference. We had __ tens and __ ones. Ten less gives us __ tens and ___ones. When we find ten less, the tens digit changes and the ones digit stays the same. *Also use known facts:* If I know ___, then I know ____.
Partitioning to subtract with regrouping The emphasis for this strategy in KS1 is to develop a deep understanding of place value.	 Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'		20—4 = 16	One ten is the same as ten ones. I know that ten minus __ is equal to ___ so I know that __ minus is equal to __.

Counting on to find the difference	 34—28 Use a bead bar or bead strings to model counting to next ten and the rest.	 Use a number line to count on to next ten and then the rest.	93—76 = 17	The __ represents the __. The __ represents the difference; it is how many more __ I need. The bigger number minus the smaller number is equal to the difference
Part Whole models – useful in examining inverse relationship with addition	Model with any concrete objects reiterating the whole and each of the parts.		Ask children to check subtraction calculations using the inverse.	Addition is the inverse of subtraction.
Using known facts	Use dienes/base ten to model the relationships. □□□ - □□= □ 	Children draw representations of H,T and O	7 – 2 = 5 so 70 – 20 = 50 so 700 – 200 = 500	I know that _ minus _ is equal to ______ so __ tens minus __ tens is equal to ___ tens.

Subtraction Year Three

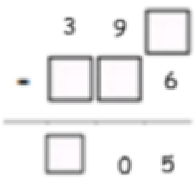
Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)
Finding the difference	Calculate the difference between 8 and 5. 	Bar models and number lines should be use to expose the structure of different forms of subtraction i.e: finding the difference, partitioning.  …and by counting on to find the difference (small difference): 231 – 198 = 33  'The difference between 198 and 231 is 33.'	8 – 5 = 3
Column subtraction without regrouping	Use base 10 or Numicon to model. Make the bigger number and then take the smaller one away.  47—32	Draw the base ten/dienes alongside the written calculation and then draw a line through the ones which have been subtracted. Draw this alongside the equation. 	47 − 24 = 23 40 + 7 − 20 + 4 20 + 3 Intermediate step may be needed to lead to clear subtraction under- standing. 32 - 12 20

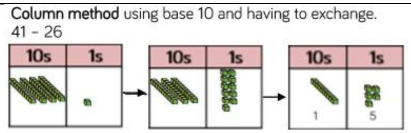
Column method with regrouping

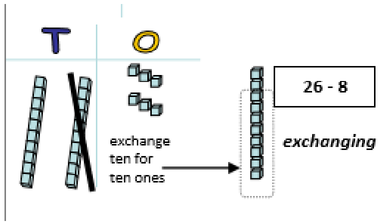
Plenty of concrete exchanging should be provided before moving to pictorial and abstract phase.

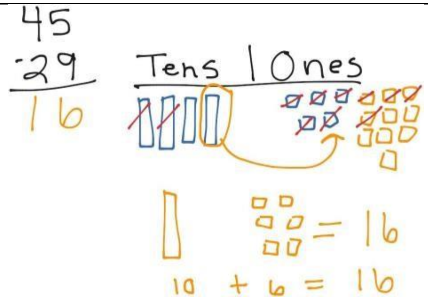
Start with one exchange before moving onto two.

Once they are confident, introduce missing digit calculations:



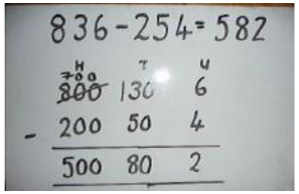




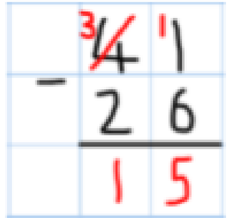


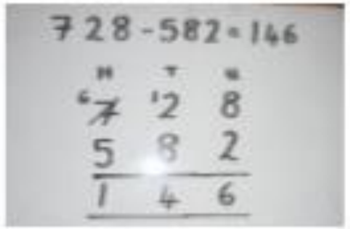
Children may draw base ten and cross off.

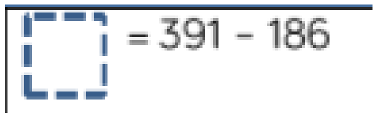
Children might again begin by partitioning the number into plave value columns:



Move on to the formal column method:







Say it (oracy)

Establishing the basics

This is a whole ____, because I have all of it.

This is not a whole ______, because I only have part of it.

A whole is always bigger than part of a whole.

A part is always smaller than its whole.

There are __ ___ in the whole group

There are ___ __ in this part of the group.

<u>When finding the difference</u>

The __ represents the __.

The __ represents the difference; it is how many more __ I need.

The bigger number minus the smaller number is equal to the difference

<u>When partitioning to subtract</u>

The __ represents all the ___.

The minus represents the ___.

The ___ represents the difference.

The difference between ___ and ____ is ___.

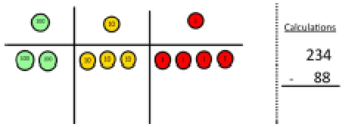
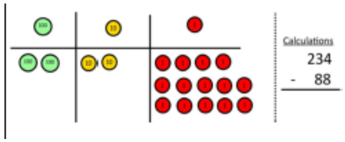
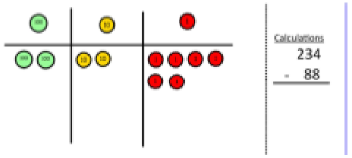
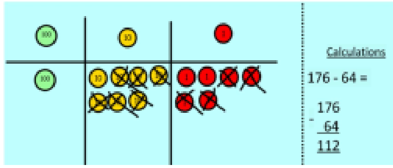
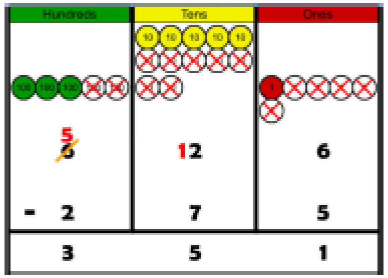
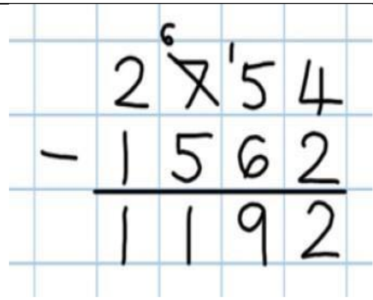
<u>When introducing column subtraction</u>

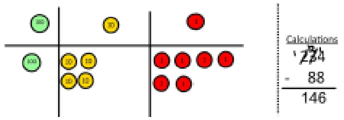
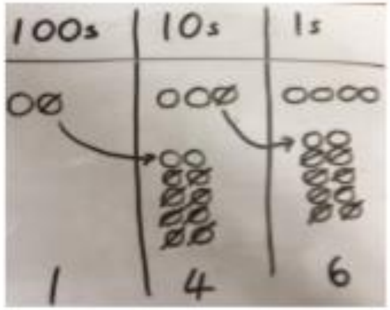
The ones column represents __ ones minus __ ones is equal to __ ones.

The tens column represents __ tens minus __ tens is equal to __ tens.

If the digit underneath is bigger than the digit above, I must exchange from the place value column from the left.

Subtraction Year Four

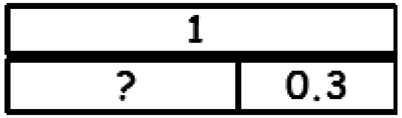
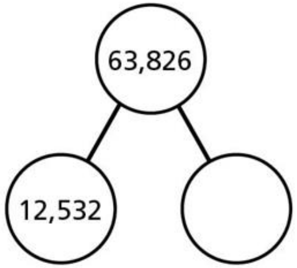
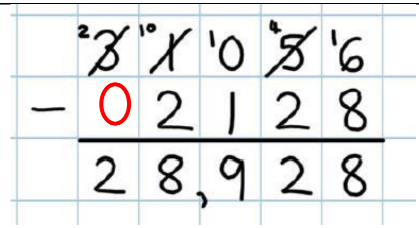
Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)
Subtracting tens and ones with up to 4 digits.	As Y3 with base ten/dienes to ensure understanding of the value of the digits, especially for regrouping. Once this is secure, progress to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters.  Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.  Now I can subtract my ones.  Now I can take away eight tens and complete my subtraction	Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.  	

	 Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		

Say it (oracy)

<u>When finding the difference</u>
The __ represents the number of children__.
The __ represents the __.
The __ represents the difference; it is how many more __ I need.
The bigger number minus the smaller number is equal to the difference
<u>When partitioning to subtract</u>
The __ represents all the ___.
The minus represents the ___.
The ___ represents the difference.
The difference between ___ and ____ is ___.
<u>When using column subtraction</u>
The ones column represents __ ones minus __ ones is equal to __ ones.
The tens column represents __ tens minus __ tens is equal to __ tens.
If the digit underneath is bigger than the digit above, I must exchange from the place value column from the left.
The bigger number minus the smaller number is equal to the difference.)
The ones column represents __ ones minus __ ones is equal to __ ones.
The tens column represents __ tens minus __ tens is equal to __ tens.
If the digit underneath is bigger than the digit above, I must exchange from the place value column from the left.

Subtraction Year Five

Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)
Subtract whole numbers with more than four digits using formal written methods. Include money and measures. Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal point.	As Y4 but consider magnitude of numbers	 Bar Model  Part/whole models See also place value counter examples above in Y4 guidance.	 Children may use zeros as place value holders.

Say it (oracy)

<u>When finding the difference</u>
The __ represents the number of children__.
The __ represents the __.
The __ represents the difference; it is how many more __ I need.
The bigger number minus the smaller number is equal to the difference
<u>When partitioning to subtract</u>
The __ represents all the ___.
The minus represents the ___.

The ___ represents the difference.

The difference between ___ and _____ is ___.

When using column subtraction

The ones column represents __ ones minus __ ones is equal to __ ones.

The tens column represents __ tens minus __ tens is equal to __ tens.

If the digit underneath is bigger than the digit above, I must exchange from the place value column from the left.

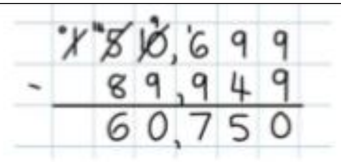
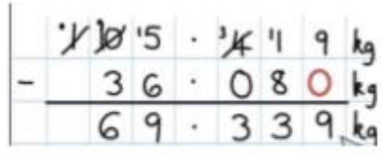
The bigger number minus the smaller number is equal to the difference.)

The ones column represents __ ones minus __ ones is equal to __ ones.

The tens column represents __ tens minus __ tens is equal to __ tens.

If the digit underneath is bigger than the digit above, I must exchange from the place value column from the left.

Subtraction Year Six

Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)	
Year 6—Subtract with increasingly large and more complex numbers and decimal values.	As Y5.	As Y5	  Ensure opportunities to subtract decimals with differing numbers of digits. e.g: 115. 01 – 2.236 = using zero as a placeholder.	

In Year Six, children should be taught to solve multi-step calculations involving more than one operation – mentally and using the written methods they have been taught – including numbers with more than four digits, decimals and in context, e.g. 3.3km, £1.99

Say it (oracy)

<u>When finding the difference</u>
The __ represents the number of children__.
The __ represents the __.
The __ represents the difference; it is how many more __ I need.
The bigger number minus the smaller number is equal to the difference
<u>When partitioning to subtract</u>
The __ represents all the ___.
The minus represents the ___.

The ___ represents the difference.

The difference between ___ and _____ is ___.

<u>When using column subtraction</u>

The ones column represents __ ones minus __ ones is equal to __ ones.

The tens column represents __ tens minus __ tens is equal to __ tens.

If the digit underneath is bigger than the digit above, I must exchange from the place value column from the left.

The bigger number minus the smaller number is equal to the difference.)

The ones column represents __ ones minus __ ones is equal to __ ones.

The tens column represents __ tens minus __ tens is equal to __ tens.

If the digit underneath is bigger than the digit above, I must exchange from the place value column from the left.

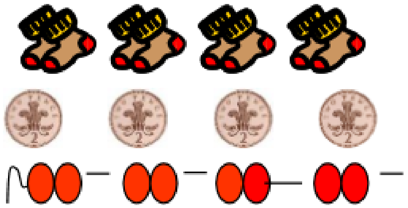
Multiplication

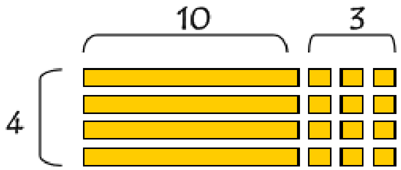
Structures of Multiplication (Haylock and Cockburn 2008)

Children should experience problems with all the different multiplication structures in a range of practical and relevant contexts e.g. money and measurement

Repeated addition

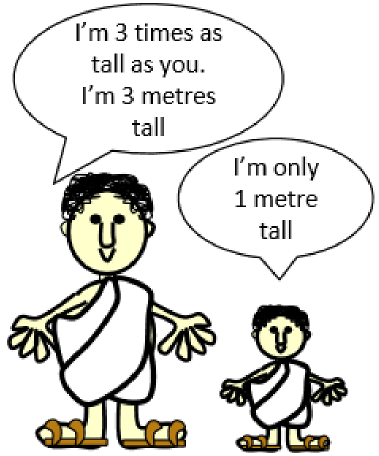
So many lots (sets) of so many
How many (how much) altogether
Per, each

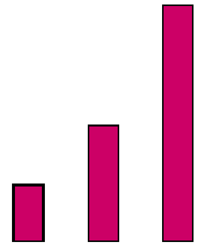




Scaling

Scaling, scale factor
Doubling, trebling
So many times bigger than (longer than, heavier than, and so on)
So many times as much as (or as many as)





scaling with Cuisenaire

Commutative law

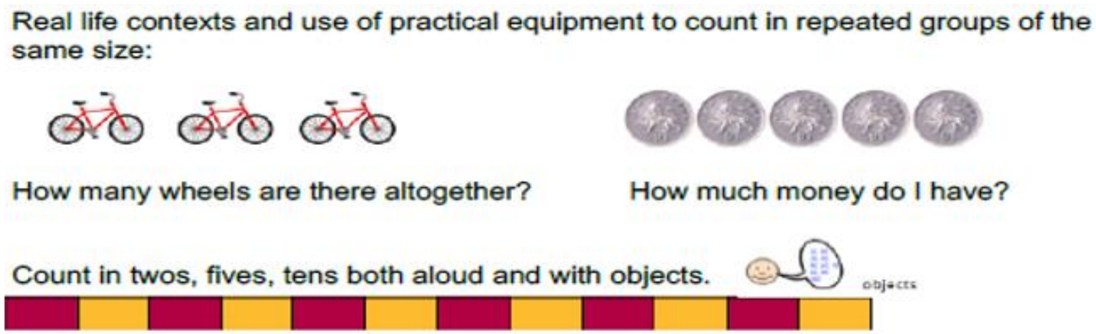
Scaling, scale factor
Doubling, trebling
So many times bigger than (longer than, heavier than, and so on)
So many times as much as (or as many as)

a x b and b x a are equal

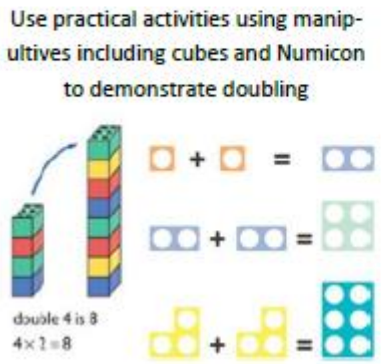
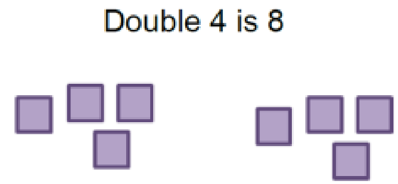
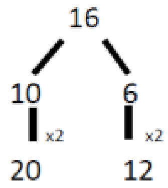
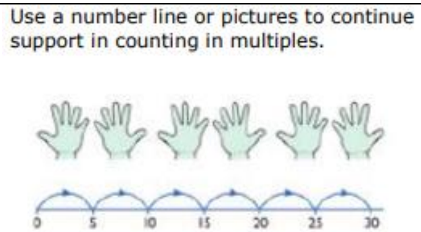
4 x 2 is the same as/equal to 2 x 4

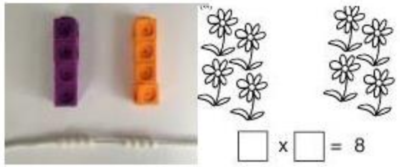
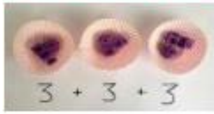
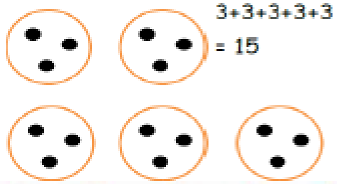
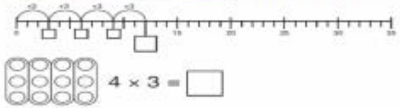
Multiplication Reception

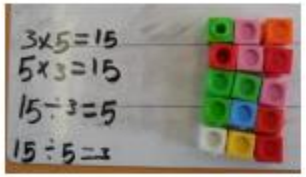
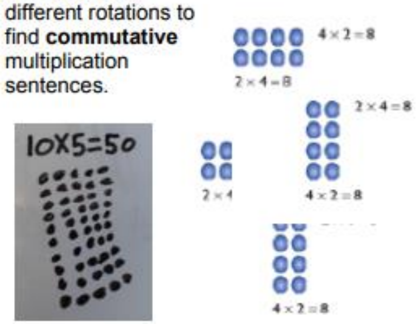
By the end of Reception, children are expected to understand the concept of doubling and to **recall some double facts within 10**. Before doubling can be introduced, children need to have a secure knowledge of counting, number facts and addition. Children are then introduced to the concept of doubling through practical games and activities, including the use of the outdoor areas. Children act out ‘doubling’ by physically adding two equal groups together to find out the ‘doubles’ answer.

Do it (concrete) / Draw it (pictorial) / Write it (abstract)	Say it (oracy)
	There are two groups of ____. I can see _______ and ______ so it must be double ____. Double _________ is __________.

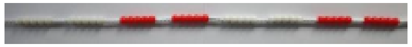
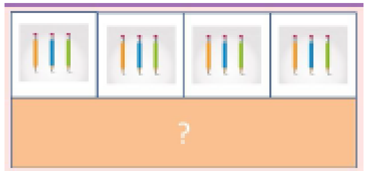
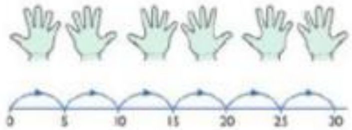
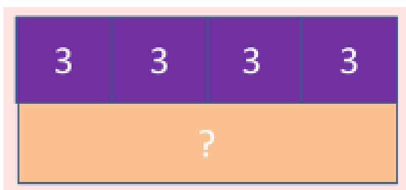
Multiplication Year One

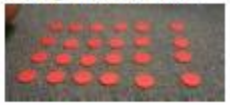
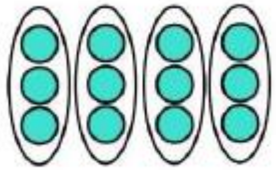
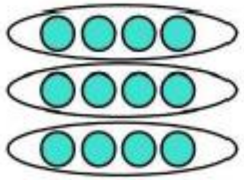
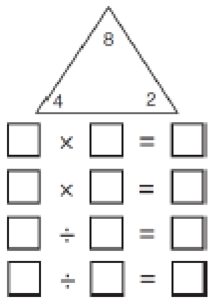
Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)	Say it (oracy)
Doubling Pupils should be encouraged to develop fluent mental recall of doubles and relate to the 2 x table.	   	Draw pictures to show how to double numbers 	 If I can see 10 wheels, how many bikes are there?	This is the same as double _. _, two times is the same as double _. Doubling a whole number always gives an even number. I know double _ is _, so two groups of _ is _. If there are two equal groups, we can use doubling facts.
Counting in multiples Pupils can use their fingers as they are skip counting, to develop an understanding of 'groups of'. Children should become increasingly fluent as they practise.	  		Count in multiples of a number aloud. Write sequences with multiples of numbers and work out missing numbers in sequences both forward and backward. If I count in 2's will I get to the number 58?	one group of ... , 2 groups of ... a times b can represent a groups of b. It can also represent b groups of a. If there are ... equal groups we can use the ... times table.

Making equal groups and counting (or skip counting) to find the total	Use manipulatives to create equal groups.  	Draw  to show 2 x 3 = 6 Draw and make representations	2 x 4 = 8	As above.
Repeated addition Pupils should apply skip counting to help find the totals of repeated additions.	Use different objects to add equal groups.  5+5+5=15  3+3+3=9	Use pictorial representations, including number lines to solve problems, e.g. There are 3 sweets in one bag. How many sweets in 5 bags?   Pupils begin to recognise the relationship between repeated addition and multiplication.  	Write addition or multiplication sentences to describe objects and pictures.  2+2+2+2+2=10 2x5=10	a+a+a is the same as a 3 times. There are __ and __ and __.... We can write this as _ plus __. plus __....

Arrays Showing commutative multiplication Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.	 	Draw arrays in different rotations to find **commutative** multiplication sentences. 	3 children go to the park to hunt for pine cones. They find 5 each, how many do they find altogether? 5 children eat the same number of cakes at a party. 15 cakes are eaten in total, how many did they each eat? 5+5+5=15 3x5=15 3+3+3+3+3=1 5 5x3=15	There are … groups of …

Multiplication Year Two

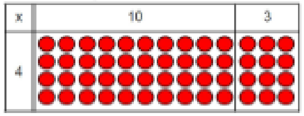
Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)	Say it (oracy)
Doubling (As Y1 but including larger 2 digit numbers.), **repeated addition and arrays.** See above.				
Count in multiples of 2, 3, 4, 5 and 10 from 0. Link skip counting to repeated addition.	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40  	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.   	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 $4 \times 3 = \square$	There are __ equal groups of __. There are __ in each group. There are __ groups of __. Factor times factor is equal to the product. The product is equal to factor times factor. __ is a factor. _ is a factor. The product of _ and _ is __. _ is the product of _ and _. If there are __ equal groups, we can use the __times table.

Multiplication is commutative	Create arrays using counters and cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	$12 = 3 \times 4$ $12 = 4 \times 3$ Use an array to write multiplication sentences and reinforce repeated addition.  $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$	
Using the inverse *Make links when teaching division, so pupils learn the relationship between the two calculations.*			$2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$ $8 = 2 \times 4$ $8 = 4 \times 2$ $2 = 8 \div 4$ $4 = 8 \div 2$ Show all 8 related fact family sentences.	

Multiplication Year Three

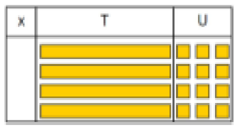
<table>
<tr><th>Strategies</th><th>Do it (concrete)</th><th>Draw it (pictorial)</th><th>Write it (abstract)</th></tr>
<tr><td colspan="4">See above for grouping, sharing, repeated addition and arrays</td></tr>
<tr>
<td>Grid Method

2-digit by 1-digit</td>
<td>Show the link with arrays to first introduce the grid method.

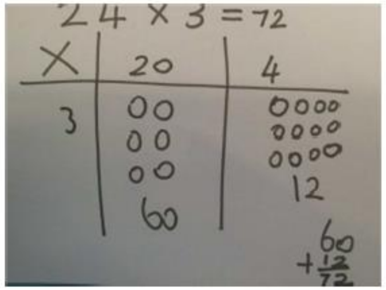


4 rows of 10
4 rows of 3

Move on to using Base 10 to move towards a more compact method.



4 rows of 13</td>
<td>Children can draw the counters or dienes.


</td>
<td>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

<table><tr><th>×</th><th>30</th><th>5</th></tr><tr><td>7</td><td>210</td><td>35</td></tr></table>
210 + 35 = 245</td>
</tr>
</table>

Say it (oracy)

Commutative law

There are __ groups of _.

There are _, two times.

a times b can represent a groups of b. It can also represent b groups of a.

Relating to times tables

If there are … equal groups we can use the ... times table.

If …. is a factor, we can use the … times table.

a+a+a is the same as a 3 times.

When using grid method

If there are ten or more ones, we must regroup the ones into tens and ones.

If there are ten or more tens, we must regroup the tens into hundreds and tens.

If there are ten or more hundreds, we must regroup the hundreds into thousands and hundreds.

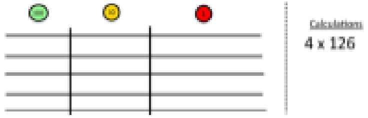
When using column method

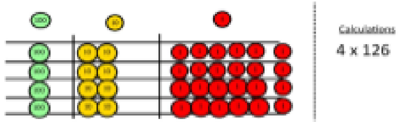
To multiply a three-digit number by a two digit number, first multiply by the ones, then multiply by the tens and then add together.

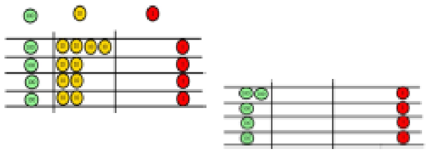
Multiplication Year Four

<table>
<tr><th>Strategies</th><th>Do it (concrete)</th><th>Draw it (pictorial)</th><th>Write it (abstract)</th></tr>
<tr><td>Grid Method

Extending Y3 method to now include 3-digit by a 1-digit number.</td><td>See Y3

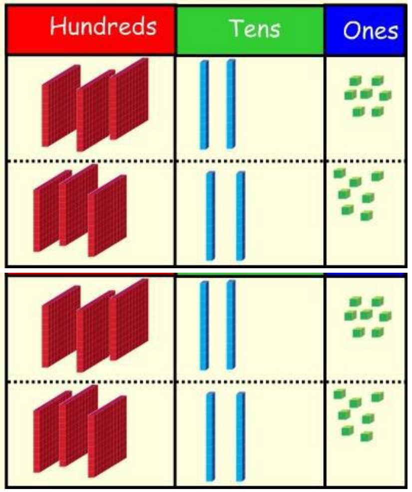
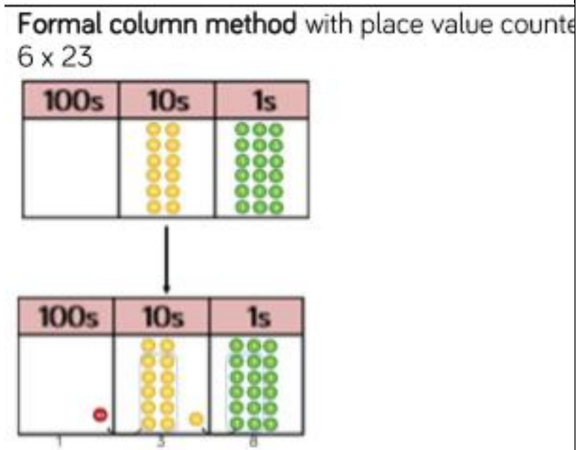
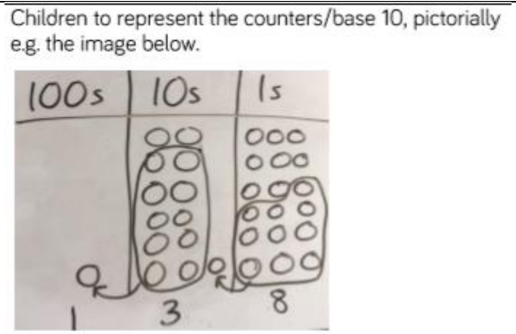
Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4

rows.

Fill each row with 126.


Add up each column, starting with the ones making any exchanges needed.


Then you have your answer.</td><td>See Y3</td><td>See Y3</td></tr>
<tr><td>Column multiplication (expanded)</td><td>Children can continue to be supported by place value counters at the stage of multiplication. Initially use examples with no regrouping, e.g. 321 x 2 = 642.</td><td>The grid method may be used to show how this relates to the formal written method.

<table><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table>
Drawing counters to show regrouping:</td><td>Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, initially, children can write out what they are solving next to their answer.</td></tr>
</table>

	 It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below. Below is an example with regrouping: 		327 x 4 28 (7x4) 80 (20 x4) 1200 (300 x4) 1308 Eventually, encourage them to stop using the brackets on the right and just write the column multiplication.

Say it (oracy)

Commutative law

There are __ groups of _.

There are _, two times.

a times b can represent a groups of b. It can also represent b groups of a.

Relating to times tables

If there are … equal groups we can use the ... times table.

If …. is a factor, we can use the … times table.

a+a+a is the same as a 3 times.

When using grid method

If there are ten or more ones, we must regroup the ones into tens and ones.

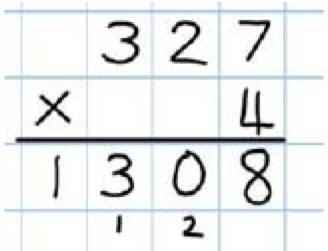
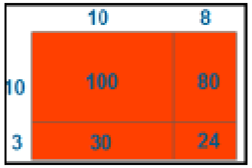
If there are ten or more tens, we must regroup the tens into hundreds and tens.

If there are ten or more hundreds, we must regroup the hundreds into thousands and hundreds.

When using column method

To multiply a three-digit number by a two digit number, first multiply by the ones, then multiply by the tens and then add together.

Multiplication Year Five

Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)
Column Method (compact) *To multiply up to 4 digits by a 1-digit number.*	See Y4 but include up to 4-digits by a 1-digit number	See Y4 but include up to 4-digits by a 1-digit number	Show how to progress from expanded/long multiplication shown in Y4 above to compact/short multiplication: 327 x 4 28 80 1200 1308 
Column Method *To multiply up to 4 digits by a 2-digit number.*	See ideas from previous years – manipulatives can still be used to model long multiplication alongside the calculation.	 Continue to use bar modelling to support problem solving.	Remind children of the importance of zero as a place value holder every time they are multiplying by tens, hundreds, thousands. 

			Expanded version above can be contracted into version below if appropriate: 1 2 3 4 × 1 6 7 4 0 4 (1234 × 6) 1 2 3 4 0 (1234 × 10) 1 9,7 4 4 Use brackets on right when introducing the method but discourage them after that.

Say it (oracy)

Commutative law

There are __ groups of _.

There are _, two times.

a times b can represent a groups of b. It can also represent b groups of a.

Relating to times tables

If there are … equal groups we can use the ... times table.

If …. is a factor, we can use the … times table.

a+a+a is the same as a 3 times.

When using grid method

If there are ten or more ones, we must regroup the ones into tens and ones.

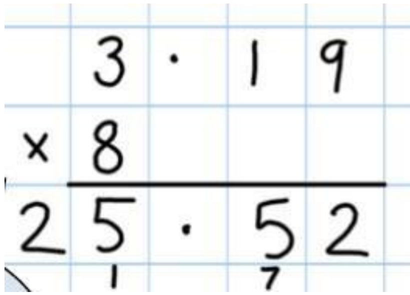
If there are ten or more tens, we must regroup the tens into hundreds and tens.

If there are ten or more hundreds, we must regroup the hundreds into thousands and hundreds.

When using column method

To multiply a three-digit number by a two digit number, first multiply by the ones, then multiply by the tens and then add together.

Multiplication Year Six

Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)
Column Method	*As Y5, using the compact method of long multiplication.*	*As Y5, using the compact method of long multiplication.*	*As Y5, using the compact method of long multiplication.*
Multiply numbers with up to 2 decimal places by whole numbers.	Model with coins. 	Use bar models to solve problems and make links to repeated addition. Short cost 5.89 each. How much will 3 pairs of shorts cost? £5.89+£5.89+£5.89=£5.89x3	There are two possible methods. **Method 1:** To solve 5.89x3, solve 589x3 using the column method and then divide the answer by 100. **Method 2:** Put the single digit in the ones column and line up the decimal points in the question and the solution: 

In Year Six, children should be taught to solve multi-step calculations involving more than one operation – mentally and using the written methods they have been taught – including numbers with more than four digits.

Say it (oracy)

<u>Commutative law</u>

There are __ groups of _.

There are _, two times.

a times b can represent a groups of b. It can also represent b groups of a.

<u>Relating to times tables</u>

If there are … equal groups we can use the ... times table.

If …. is a factor, we can use the … times table.

a+a+a is the same as a 3 times.

<u>When using grid method</u>

If there are ten or more ones, we must regroup the ones into tens and ones.

If there are ten or more tens, we must regroup the tens into hundreds and tens.

If there are ten or more hundreds, we must regroup the hundreds into thousands and hundreds.

<u>When using column method</u>

To multiply a three-digit number by a two digit number, first multiply by the ones, then multiply by the tens and then add together.

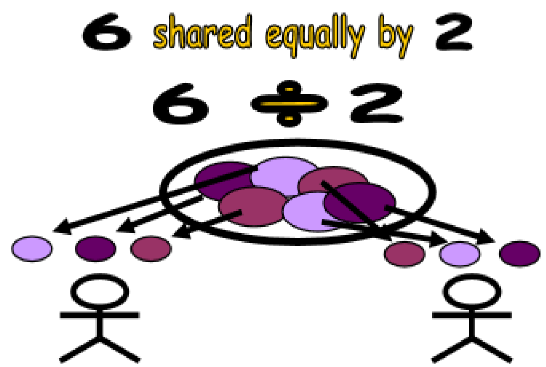
Division

Structures for Division (Haylock and Cockburn 2008)

Children should experience problems with the different division structures in a range of practical and relevant contexts e.g. money and measurement

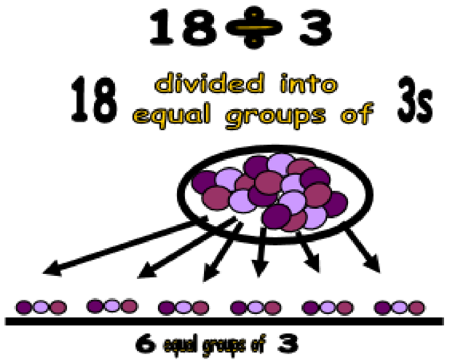
Equal-sharing

Sharing equally between
How many (much) each?

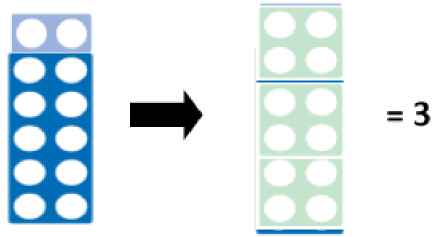


Inverse of multiplication (Grouping)

So many lots (sets/groups) of so many
Share equally in to groups of …



Divide twelve into equal groups of four



Make 12

Overlay groups of four

Ratio structure

comparison
inverse of scaling structure of multiplication
scale factor (decrease)

Barney earns three times more than Fred. If Barney earns £900 how much does Fred earn?

Jo's journey to school is three times as long as Ella's. If Jo walks to school in 30 minutes how long does it take Ella?

Division Reception

There are no division objectives in the EYFS statutory framework. However, children need to have a secure knowledge of counting backwards, number facts and subtraction. Children may then be introduced to the concept of halving and sharing through practical games and activities. They may act out 'halving and sharing' through activities such as sharing food for their Teddy Bear's Picnic, sharing resources equally to play a game. This is reinforced by opportunities provided in the outdoor area for the children to halve and share out objects such as building blocks, twigs etc.

Do it (concrete) / Draw it (pictorial) / Write it (abstract)




Division can be introduced through halving.

Children begin with mostly pictorial representations linked to real life contexts.

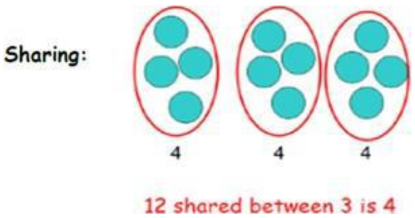
Mum has 6 socks. She grouped them into pairs – how many pairs did she make? How many socks did she have altogether?

Sharing model:
I have 10 sweets. I want to share them with my friend.
How many will we have each?

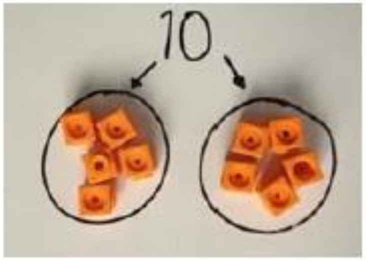
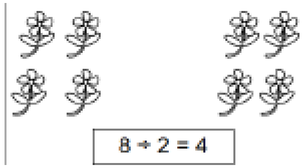
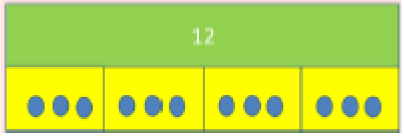
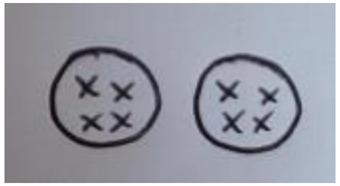
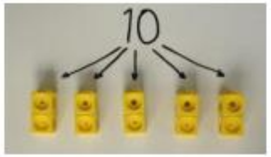
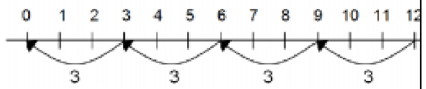
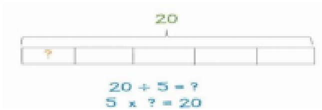
Say it (oracy)

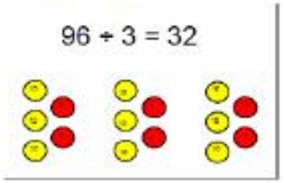
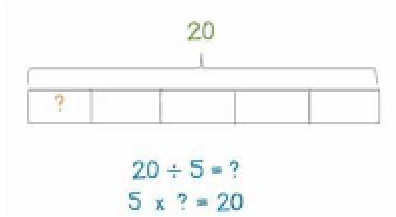
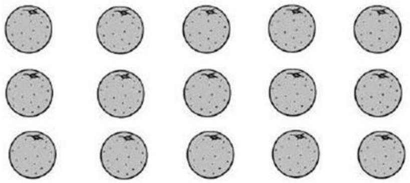
There are _ altogether; half of _ is equal to .
Half of _ is equal to _.
I know that double _ is _; so half of _ is _.

Division Year One

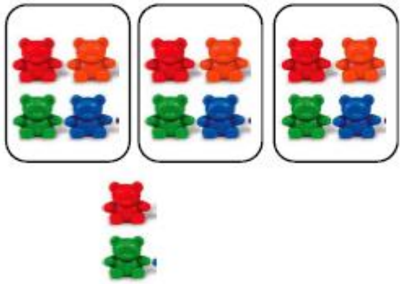
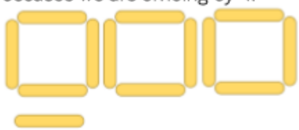
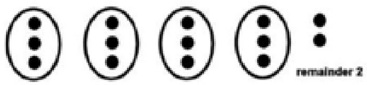
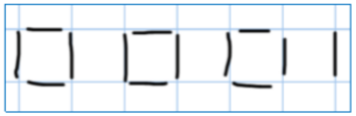
Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)	Say it (oracy)
Sharing Here, division is shown as sharing. E.g. If we have 24 squares of chocolate and we share them between 3 people, each person will have 8 squares each.	   	Children use pictures to share quantities, e.g. 8 shared between 2 is 4:  	Share 20 buns between five people. 20 ÷ 5 = 4 Can you make up your own 'sharing' story and record a matching equation?	We can represent this as _ divided between _. _ divided between _ is equal to _ each.

Division Year Two

Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)	Say it (oracy)
Division as sharing	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  Children use bar modelling to show and support understanding.  12 ÷ 4 = 3 Draw circles and crosses, e.g. 8 ÷ 2 = 4 	12 ÷ 3 = 4	If __ is a factor, we can use the __times table. __ is the dividend. __ is the divisor. __ is the quotient. We can skip count using the divisor to find the quotient.
Division as grouping Here, division is shown as grouping. If we have ten cubes and put them into groups of two, there are 5 groups. This is a good opportunity to demonstrate and reinforce the inverse	 Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.	Show jumps in groups. The number of jumps equals the number of groups.   Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group? Max is filling party bags with sweets. He has 20 sweets altogether and decides to put 5 in	_ is divided into groups of _. There are _ groups. _ is divided into__ groups of ___.

relationship with multiplication.	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4  	Continue to use bar modelling to aid solving division problems.  	every bag. How many bags can he fill? How many groups of 6 in 24? $24 \div 6 = 4$	The _ represents the total number of _. The _ represents the number of _ in each group.
Division within arrays Use arrays of concrete manipulatives and images of familiar objects to find division equations. Begin to use dot arrays to develop a more abstract concept of division.	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$	Draw an array and use lines to split the array into groups to make multiplication and division sentences. 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$	There are … groups of …

Division Year Three

Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)
Use sharing, grouping and arrays as in KS1.			
Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over.  Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over.	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. 13 ÷ 4 = 3r1  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders.  	Complete written divisions and show the remainder using r: 13 ÷ 4 = 3r1 

Say it (oracy)

<u>Getting to grips with the basics</u>

_ divided between _ is equal to _ each.

_ is divided into groups of _. There are _ groups.

The _ represents the total number of _. The _ represents the number of _ in each group.

There are … groups of …

<u>Division with remainders</u>

_ divided into groups of _ is equal to _, with a remainder of _.

The largest multiple of _ that is less than or equal to _ is _.

The remainder is always less than the divisor.

_is a multiple of _, so when it is divided into groups of _ there are none left over; there is no remainder.

_is not a multiple of _, so when it is divided into groups of _ there are some left over; there is a remainder.

If the dividend is a multiple of the divisor, there is no remainder.

If the dividend is not a multiple of the divisor, there is a remainder.

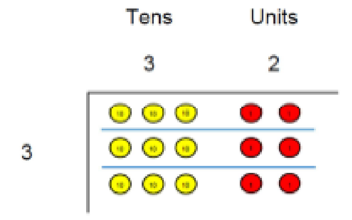
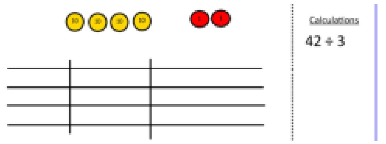
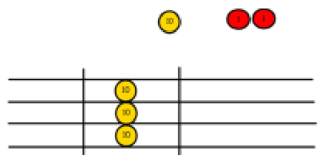
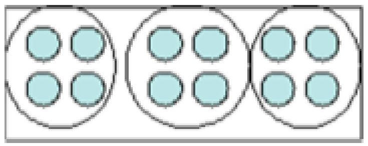
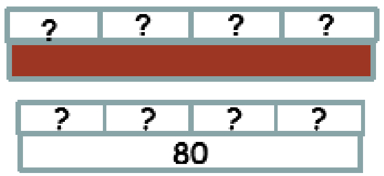
<u>Short division</u>

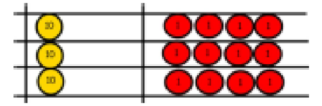
If dividing the tens gives a remainder of one or more tens,

we must exchange the remaining tens for ones.

If dividing the hundreds gives a remainder of one or more hundreds, we must exchange the remaining hundreds for tens.

Division Years Four and Five

Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)
Short division	Use place value counters to divide using the bus stop method alongside  Use place value counters to divide using the bus stop method alongside.  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.  bar models	Begin with divisions that divide equally with no remainder. 2 1 8 3 4 | 8 7 2 Move onto divisions with a remainder. 8 6 r 2 3 5 | 4 3 2 Finally move into decimal places to divide the total accurately. 1 4 . 6 16 21 3 5 | 5 1 1 . 0

	 We look how much in 1 group so the answer is 14.		

Say it (oracy)

Getting to grips with the basics

_ divided between _ is equal to _ each.

_ is divided into groups of _. There are _ groups.

The _ represents the total number of _. The _ represents the number of _ in each group.

There are … groups of …

Division with remainders

_ divided into groups of _ is equal to _, with a remainder of _.

The largest multiple of _ that is less than or equal to _ is _.

The remainder is always less than the divisor.

_is a multiple of _, so when it is divided into groups of _ there are none left over; there is no remainder.

_is not a multiple of _, so when it is divided into groups of _ there are some left over; there is a remainder.

If the dividend is a multiple of the divisor, there is no remainder.

If the dividend is not a multiple of the divisor, there is a remainder.

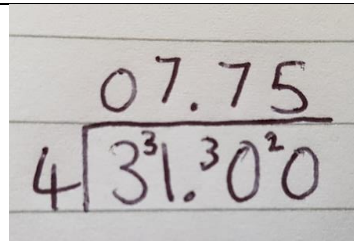
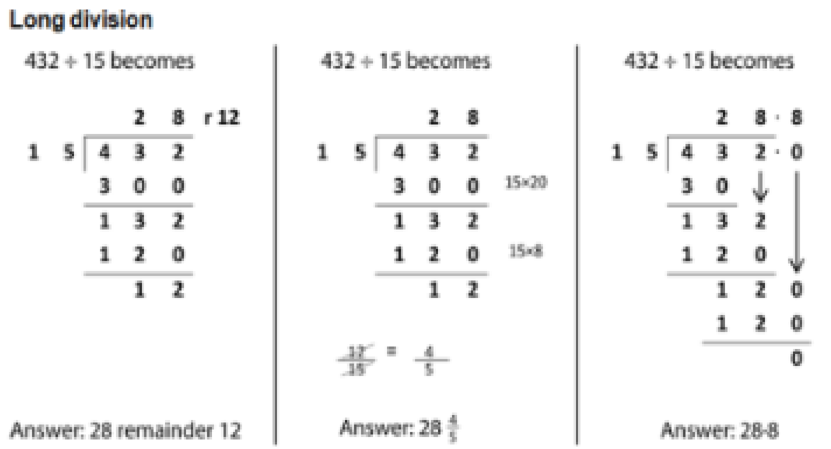
Short division

If dividing the tens gives a remainder of one or more tens,

we must exchange the remaining tens for ones.

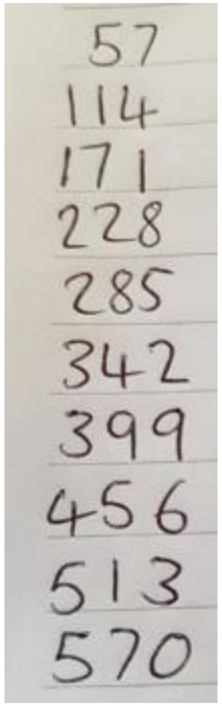
If dividing the hundreds gives a remainder of one or more hundreds, we must exchange the remaining hundreds for tens.

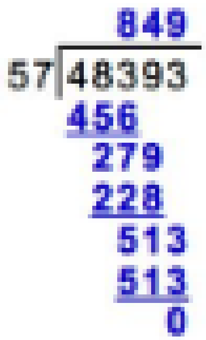
Division Year Six

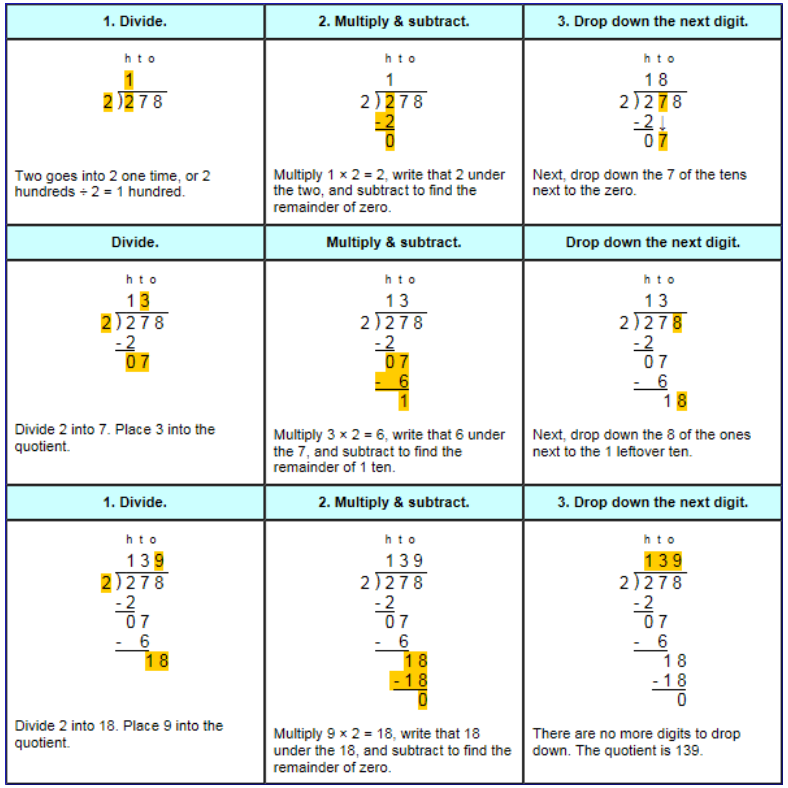
Strategies	Do it (concrete)	Draw it (pictorial)	Write it (abstract)
Short division *As Y4/5 but interpret extend to cases where the answer has up to two decimal places. Also interpret remainders as whole numbers, fractions or by rounding.*	*See Y4/5* For decimals, use example of money, e.g. four people spend £31 at a restaurant, how can they split the bill equally? For fractions, use items which can be cut equally or fractions resources, e.g. 3 people want to share 7 cakes. How can they do it fairly with no cake left over?	Draw the items being divided and consider how any remainder can be drawn and then divided.	 so £31÷4 =£7.75 $19 \div 5 = 3\text{ R }4 = 3\frac{4}{5}$ Remainder
Long division *Concrete and pictorial representations can be similar to those used for short division. There are two possible methods. Choose one and stick with it.* *Children need to be able to interpret remainders as whole*	*When dividing by a two-digit number, before beginning the calculation, it may be helpful to write out at least the first five multiples of that number to assist the calculation. If it is helpful, they may write all ten multiples.* **Method 1, including place value holders:**		

numbers, fractions and decimals.

Method 2, without place value holders:







1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
Divide 2 into 7. Place 3 into the quotient.	Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	**2. Multiply & subtract.**	**3. Drop down the next digit.**
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	There are no more digits to drop down. The quotient is 139.

In Year Six, children should be taught to solve multi-step calculations involving more than one operation – mentally and using the written methods they have been taught – including numbers with more than four digits.

Say it (oracy)

Getting to grips with the basics

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The _ represents the total number of _. The _ represents the number of _ in each group.

There are … groups of …

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The largest multiple of _ that is less than or equal to _ is _.

The remainder is always less than the divisor.
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If the dividend is a multiple of the divisor, there is no remainder.
If the dividend is not a multiple of the divisor, there is a remainder.

Short division

If dividing the tens gives a remainder of one or more tens,
we must exchange the remaining tens for ones.
If dividing the hundreds gives a remainder of one or more hundreds, we must exchange the remaining hundreds for tens.

Appendix: Fluency progression – Expected Number Facts